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AN ANALYSIS OF THE PROBLEM-SOLVING-BASED TEACHING METHODOLOGY IN THE APPROACH TO GEOMETRY

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ABSTRACT
This article discusses the teaching of geometry through problem-solving as a methodology, characterizing itself as a literature review of authors who deliberate on the subject. Thus, this investigation aims to understand how problem-solving can contribute as a methodology for approaching geometry in the classroom context. The strategy used for data collection was an in-depth study of the literature, in order to confront different points of view and thus consolidate considerations on the subject. At the end of the research, it is observed that this offers support in terms of learning and improving the teaching processes of Mathematics and, above all, Geometry.

Keywords: Problem solving; Methodology; Teaching; Geometry.

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INTRODUCTION

We are embedded in a world filled with shapes that are, in most cases, not fully understood in their breadth. The notion of the space occupied by a body gives rise to significant discussions about its existence. Estimating the volume of a geometric solid, as well as analyzing its dimensions and the applicability of forms in daily life, is not a recent human need. When we examine the history of mathematics and the evolution of humankind in terms of understanding, we notice that, as societies advanced, problems related to geometric aspects appeared with increasing frequency.

Within this scenario, practical needs spurred the formulation of situations in which spatial geometry was present, as well as the difficulties in dealing with such concepts. At other times, the sheer beauty that this knowledge could provide was sufficient to inspire the creation of such situations. The quest for advances in mathematical knowledge led to the development of ideas and methods that traverse paths where the completeness of processes permeates their complexity, without, however, revoking their irrefutable relevance and applicability both in mathematical procedures and in the representation of the reality around us. Thus, this teaching work bears the following theme: an analysis of the problem-solving-based teaching methodology in the approach to geometry.

Based on the teaching of geometry in a general context, as well as on problem solving related to the theme, the study seeks to answer the following research question: in what ways can problem-solving practice be applied to the teaching of geometry in basic education? We began with the hypothesis that, in addition to reducing the arguments and artifices necessary for the deduction and use of formulas related to the geometric context, problem solving assists in constructing and developing spatial reasoning and the capacity to associate geometric concepts with realistic situations.

The main purpose of this work is to understand how the teaching of geometry occurs in basic education and how problem solving can be applied to the approach to this field of mathematics. Specifically regarding the development of the study, the initial intention is to understand problem solving as a teaching methodology at the elementary level, correlating the practice with the teaching of geometry.

The relevance of this work may be considered undeniably indispensable, given that understanding concepts related to geometry and to problem solving begins in basic education, still in elementary school, and is deepened in high school. However, both the teaching of this field of knowledge in a general context and that of mathematics are tied to a range of abstractions and cognitive perceptions that, in most cases, cause irreversible harm to the learner's education when they are not well defined and worked through. Thus, it is necessary to seek strategies that facilitate the teaching of such concepts, and problem solving is grounded as a crucial tool for understanding geometric concepts—which gives significant importance to the need for its understanding even within the academic environment.

The development of this work took place using bibliographic research processes with qualitative emphases. For data collection, we propose a literature review addressing the topic in order to characterize a qualitative parallel among the different conceptions on the matter. This research, in turn, is grounded in various authors, both from applied mathematics and from mathematics education. Among them we highlight: Boyer, Carl B. (História da matemática); PAVANELLO, R. M., "O abandono do ensino de geometria no Brasil: causas e consequências"; LORENZATO, S., "Por que não ensinar Geometria?" Revista da Sociedade Brasileira de Educação Matemática; FAINGUELERNT, E. K., "O Ensino de Geometria no 1º e 2º graus." Revista da Sociedade Brasileira de Educação Matemática; DANTE, Luiz Roberto, Formulação e resolução de problemas de matemática, among others.

This work is divided into three fronts of discussion. First, we discuss the practice of problem solving from the perspective of the mathematics teacher. Second, we present an analysis of the phases of problem solving through the lens of the teacher as a mediator. Third and finally, we highlight possibilities in teaching practice for the teaching of geometry through problem solving.

METHODOLOGY

THE MATHEMATICS TEACHER AND INSTRUCTION ORIENTED TOWARD PROBLEM SOLVING

In the exercise of their profession, mathematics teachers face various barriers that challenge and deconstruct their identity. Devaluation, lack of professional preparation, and lack of school infrastructure are among the main problems that afflict the educator's work. However, the principal difficulty faced by education today is the student's blockage with respect to mathematics as a subject—a blockage that has been the result of doctrines and traditional teaching practices that place the student in the background. The consequences of this blockage are reflected in a lack of motivation, disinterest, school dropout, and the high rates of illiteracy and poverty in the country. Aware of the reality of Brazilian education, the main challenge of contemporary education is to bring young people back to classrooms, showing them the possibilities that quality education can afford. It is necessary that students become the fruit of their own knowledge and learn from their experiences, as Freire asserts:

> "Education is a response from finitude to infinitude. Education is possible for human beings because they are unfinished and know themselves to be unfinished. This leads them toward their perfection. Education, therefore, implies a search carried out by a subject who is the human being. 'The human being must be the subject of their own education.'" (Freire, 1985, p. 14).

From the understanding that we are in a constant process of evolution and refinement, knowledge is forged. Education, in turn, is the fruit of that understanding, and from the young person must come the

comprehension of their role vis-à-vis the needs of the world that surrounds them. It is no different with mathematics. Mathematical concepts—together with the geometric concepts that form the subject of this work—act directly and decisively in representing reality and, as such, should be explored in their full breadth. Nonetheless, the traditional pedagogy—which is still present today in some Brazilian schools—fails to fulfill this role, resulting in the learning deficits found across various levels of schooling.

Reformulating the instruction developed in public schools is the only way to change this reality that afflicts contemporary education. Here we propose instruction based on the problem-solving methodology as a way out of the aforementioned problems. In most cases, this methodology has succeeded in drawing students' attention to mathematics education insofar as it proves challenging and an attractive way to represent a mathematical situation. Dante argues that:

> "Rapid social changes and the ever-greater and faster enhancement of technology prevent us from making an exact prediction of which mathematical skills, concepts, and algorithms would be useful today to prepare a student for their future life. Teaching only those concepts and algorithms that are currently relevant does not seem to be the way, because they may become obsolete in fifteen or twenty years, when today's child is at the height of their productive life. Thus, a quite reasonable path is to prepare the student to deal with new situations, whatever they may be. For this reason, it is essential to develop initiative, an exploratory spirit, creativity, and independence through problem solving." (Dante, 2007, p. 12).

It is evident that a teaching methodology based on the mere mechanical transmission of knowledge is no longer a viable route to meet the current demands of society. Mathematically speaking, instruction based on concepts, formulas, and procedures with high rigor and complexity is not the path to achieving education of social relevance and quality—education that can represent the real world in which the student is embedded. Problem solving can contribute significantly to changing this reality, since it can directly support the teaching-learning process.

Speaking of "problem" in the context of mathematics may at first cause some apprehension, given that we are proposing here a methodology capable of bringing students back to the school environment. Nevertheless, drawing on studies by different researchers in the field, one can see that this methodology truly has an effect in pursuing this objective. Solving problem situations in the classroom elevates students to a higher level of intellectual and cognitive development, since such situations allow the refinement of mental structures and confront learners—directly and indirectly—with their reality. According to Dante:

> "A mathematics class in which students, encouraged and guided by the teacher, work actively—individually or in small groups—on the adventure of seeking the solution to a problem that challenges them is more dynamic and motivating than one that follows the classic scheme of explanation and repetition. The real pleasure of studying mathematics lies in the satisfaction that arises when the student, on their own, solves a problem. The more difficult it is, the greater the satisfaction in solving it. A good problem arouses curiosity and triggers in the student a research behavior, reducing their passivity and conformism." (Dante, 2007, pp. 13–14).

For Dante, when a teacher develops instructional work from the perspective of problem solving, they offer the student the possibility of ceasing to be a passive subject—one who is merely present to receive information and copy pre-established patterns of thought—and becoming an active subject who will be the protagonist in constructing their own knowledge. Dante further states that problem situations in the classroom contribute significantly to energizing teachers' work, as well as to motivating students regarding mathematical knowledge.

With meaningful and transformative learning as the principal focus, it is necessary to understand that problem solving, in and of itself, does not guarantee success in achieving such an objective. This methodology must be accompanied by factors that substantiate its application. One fundamental factor in this perspective is the development of sound planning that addresses the different challenges encountered in the classroom, as well as possible avenues for resolution. It is a fact that, in a single school environment, we observe the presence of children with different levels of learning—some with relative ease in developing the proposed activities, others with attention deficits and difficulties learning the content covered, especially in mathematics. The teacher's planning of didactic situations involving problem solving to be experienced in the classroom must include strategies that address the different learning conditions encountered and that guarantee equitable instruction for all.

Still according to Dante, among the skills to be developed in individuals through this teaching methodology is the capacity to develop in them the conditions necessary to identify and use their own resources to solve real-world problems—those present both inside and outside the school environment. He reaffirms this when he states that: "[…] it is necessary to develop in the student the ability to make a logical argument and to make intelligent and effective use of available resources, so that they can propose good solutions to the issues that arise in their daily lives, at school or outside it." (Dante, 2007, p. 11).

More than merely proposing a problem for students to solve, the teacher must be able to identify which problems are relevant to be discussed in the school environment. This is the second factor to be considered when working with this methodology. A problem that is aligned with current curriculum proposals should be a challenging situation—one that leads the student to think and to seek the best strategy for solving it. Fundamentally, it must also have a context oriented toward reality, which is so often discussed as the principal stumbling block for mathematics teachers today. In other words, it is necessary to discuss situations that motivate the student to take a stand in relation to the solution, placing

themselves fully within the problem at hand. Only in this way will students develop the habit of questioning what they are learning, setting aside learning based merely on repeating what the teacher is doing. As Ana Lúcia Braz Dias states:

> "The great objective of the school is to prepare the student to solve problematic situations that they encounter in their daily life and that they will encounter in their adult life. It is expected that each area of school learning will contribute to this objective. Mathematics can also contribute to the resolution of problematic situations. For example, it is certain that the knowledge built about numbers and operations, about shapes, about measurements, and about the organization and interpretation of quantitative information may be necessary in this task." (Gestar II – Matemática, MEC, 2009).

For Dias, the problem situations proposed by the teacher in the classroom—which she calls "problematic situations"—must align with those that students will likely face in their adult life; that is, the student must be able to interpret, give meaning to the solution, relate it to daily life, and select means for its resolution. The posture of the teacher as the owner of knowledge and truth—the figure who transmits knowledge—must be set aside, making room for the teacher as mediator, whose role will be to assist the student in constructing knowledge. This raises the question: how should one work with problem solving in the mathematics classroom? This is what we will now address.

MAKING MATHEMATICS TEACHING DYNAMIC: A LOOK AT THE PHASES OF PROBLEM SOLVING AND MATHEMATICS INSTRUCTION FROM THE STANDPOINT OF A TEACHER-MEDIATOR

Instruction from the perspective of problem solving can be an important tool for the student's intellectual development and for overcoming difficulties related to learning mathematics. However, it also becomes evident that this practice will only be effective if accompanied by good planning and by a change in the teacher's posture regarding their role in the classroom. Up to this point, we have discussed the practice of problem solving and its applicability in Brazilian schools. But, after all, what is a problem?

According to Silveira Bueno's Novo Dicionário da Língua Portuguesa: "problem /ê/ [from Late Latin problema] s.m. 1. Question, difficulty to resolve. 2. That which is difficult to explain." Nonetheless, problem solving in the classroom is not based on proposing questions with a high degree of complexity and difficulty of understanding, since such questions discourage students and hinder their learning. Here we discuss a teaching methodology whose main objective is to support the teacher's work in the teaching of geometry by means of problems that are significant to students' engagement with the world around them. To work with this practice, however, it is necessary to understand the essential elements in its development—that is, the steps for solving a problem. According to the National Curriculum Parameters (Parâmetros Curriculares Nacionais, PCNs):

> "Problem solving, in the perspective indicated by mathematics educators, enables students to mobilize knowledge and develop the capacity to manage the information at their disposal. Thus, students will be more likely to increase their knowledge of concepts and procedures as well as their view of problems, of mathematics, and of the world as a whole, developing self-confidence about this learning." (PCNs, 2007, p. 40).

Students employ a range of strategies to solve mathematical problems; there are also many types of problems to analyze. The most common are open problems and those that involve several steps before reaching a solution. Real problems, however, are the ones that best represent reality, since they incorporate, in their structure, more than a question or a challenge: an appropriate contextualization. By understanding mathematical problem solving, students begin to use mathematical concepts meaningfully and correctly to seek strategies that lead to a satisfactory result. Upon achieving this result, the student comes to understand their own potential and is spurred by the satisfaction of learning to forge their identity.

We come to understand that a problem exists from the moment there is a need to achieve a common objective—what we call the solution. In mathematics, based on conceptions of problems, we understand the true meaning of the educator's importance in carrying out quality work. When these same problems are formulated by the educator from the perspective of a broader context representing the student's daily life, we call them problem situations (situações-problema). In parallel with the practical meaning of learning, problem situations act on the student's cognitive structures, offering great challenges and potentialities. Therefore, problem solving—much more than a method that aids the construction of mathematical concepts—also helps students' intellectual development. Dante writes:

> "Problem situations are application problems that portray real, day-to-day situations and that require the use of mathematics to be solved. Through mathematical concepts, techniques, and procedures, one seeks to mathematize a real situation by organizing data in tables, drawing graphs, performing calculations, etc. In general, these are problems that require research and data collection. They can be presented in the form of projects to be developed using knowledge and principles from other areas besides mathematics, provided that the answer relates to something that arouses interest." (Dante, 2003, p. 20).

Mathematics instruction mediated by problem solving becomes much more interesting when the educator uses good exercises that stimulate students' learning and curiosity instead of relying on simple repetitive activities—practices that have persisted since the traditional school and that lead to tedious reproductions of drill exercises, thereby contributing to the eventual distancing from the student's reality. Study based on problem solving, however, should follow certain stages that ensure understanding of this type of activity and facilitate students' search for a solution. These are the stages we now describe.

The first step in solving a problem is its **comprehension**. At this stage, the student will ask: what is the proposed problem? What are the data provided by the question, and what do they indicate? Are the

data provided sufficient or insufficient with respect to what is being asked? In view of these questions, the student can take a position regarding the situation and prepare for the next stage, which is called **devising a plan of action**.

Here, the student will build bridges between the information provided and the problem proper, which, in some moments, we may call the unknown. It is of fundamental importance to characterize the situation under analysis, asking about its connections with mathematics and its tools. It is crucial for the student to consider whether they have seen the problem in previously experienced situations—even if under different circumstances. Perceiving these characteristics will help identify which theorems, properties, or conceptual fields of mathematical knowledge will be useful when seeking a solution.

At this stage of resolution, it is common for the first difficulties to appear, mainly related to the information provided. However, if the student is unable to solve the proposed problem immediately, the educator can broaden the range of strategies by proposing that the student first solve a problem that is connected to the context of the situation addressed, or by specifying a component of the problem that can be solved first; as Dias states:

> "It is clear that the knowledge available to the person attempting to solve a problem has considerable influence on the success of obtaining a solution. This includes both mathematical knowledge and extra-mathematical knowledge related to the problem. Knowledge of similar problems and of strategies used to solve other problems also increases the chances of success." (Gestar II – Matemática, MEC, 2009, p. 50).

In this sense, the importance of the knowledge acquired by the student for solving tasks of this type—those proposed by the educator—becomes clear. Thus, problem solving not only develops mathematical knowledge in its fullness, it also contributes to establishing links between mathematics and other fields of knowledge. Once the stages of comprehension and plan-setting that will guide the resolution process are completed, it is time to put the plan into action—that is, the moment of execution arrives.

In the third stage, called **executing the plan**, the student defends their arguments (in this case, the mathematical knowledge at their disposal) in practice. Doubts arise with greater force, since, once the strategy has been outlined, most students face difficulties in manipulating the tools of knowledge and applying them to the appropriate contexts. From the perspective of the traditional teacher, the response is, on many occasions, to indicate the answer to the student, thereby blocking their cognitive capacities for reasoning and perception. From the perspective of the teacher-mediator, a bridge is established between knowledge and the student, who will be encouraged to draw their own conclusions and construct their answer to the proposed problem.

The difference lies in how the student is guided to reach the result. In the first case, the traditional teacher will "complete" the student's answer, failing to provide the conditions for the student to do so themselves. In the second case, the teacher-mediator will work precisely to explore each student's potential, enabling them to surpass their own limits and achieve transformative learning. According to Oliveira, the manner in which the teacher explores the problem is also a fundamental point to be discussed. For him:

> "For the student to be able to read and understand the problem, it is useful that, during class, the problems be explored orally, working through different ways of finding the solution. It is worth remembering that it is also important to work with problems involving the student's daily life so as to make them more interesting." (Carvalho, 2007, p. 18).

The fourth and final step in problem solving—often forgotten by students and by mathematics teachers—is verification or **retrospection**. At this stage, the student verifies the result obtained as well as the reasoning employed. It is desirable that, at this stage, the student perceives whether it would be possible to reach the same result by different paths. This awareness contributes significantly to understanding that, in solving mathematical problems, there are multiple paths that can be used to arrive at the same result.

FROM THEORY TO PRACTICE—ANALYZING POSSIBILITIES IN TEACHING PRACTICE FOR THE TEACHING OF GEOMETRY THROUGH PROBLEM SOLVING

As seen previously, pedagogical work through problem solving is not merely a simple teaching methodology; rather, it is a proposal that involves the interaction of diverse mathematical concepts in activities developed in the classroom with the world beyond the classroom's limits. This is not easy work. To this end, it is of fundamental importance that the teacher, as mediator of learning, interact with everyday situations that are part of each student's reality.

The educator must bear in mind the importance of learning new content through mathematical problem solving. However, for this learning to be consolidated, the student must possess some mathematical knowledge enabling them to find paths that lead to diverse strategies in the problem-solving process. In teaching geometry, students should be guided to read and correctly interpret the information contained in the proposed activities; understanding the basic components used to reach the solution of the problem—which were discussed earlier—is paramount.

Amid the neglect of geometry teaching discussed earlier—whether due to school systems or to educators themselves—the fact remains that this branch of mathematical knowledge establishes strong connections with the student's intellectual development, helping to develop mental perceptions and the

representation of the space in which the student is situated. In this sense, geometry in the school context should provide means by which students can establish representations of the real world, and its approach in the final years of elementary education should create opportunities for the first encounters with more developed and systematized thinking. According to Lorenzato:

> "Geometry studies from the 5th to the 8th grade should provide opportunities for students to carry out their first systematic explorations. It is at this stage that the first logical deductions are constructed; the results and processes should be discussed, though without concern for formalization." (Lorenzato, 1995, p. 8).

For him, at this stage the student needs to understand the process that made the result possible, leaving formalizations and rigor in developing the proposed problems to a secondary plane. In other words, the principal concern of the mathematics educator when presenting geometric concepts at this moment of learning should be directed toward discussing the understanding and meaning of the paths used by the student in order to arrive at a result and promote learning. Applying problem situations that prompt students to use strategic mechanisms of geometry and that contribute to content development becomes essential.

In geometry, problem solving enables the student to think critically about their learning, developing logical reasoning amid external representations and demonstrating good and creative solutions to the proposed problems—including those directly related to daily life. This teaching methodology also plays a fundamental role in identifying students' weaknesses related to mathematical knowledge, helping to resolve doubts that have persisted over the years and that traditional teaching fails to eliminate. By awakening in the student an interest in and curiosity about learning mathematics, the problem-solving methodology promotes an approximation between young people and school—or between young people and knowledge itself—especially with regard to geometric thinking. According to Dias:

> "Teaching via problem solving means considering the problem as a triggering element of a process for constructing mathematical knowledge. That is, problems aim to contribute to the formation of concepts even before their presentation in mathematical language. It is the need to solve the problem that leads the student to appropriate—alone or collectively—the intellectual tools necessary to construct a solution." (Gestar II – Matemática, MEC, 2009, p. 51).

Engaging students in situations that motivate them to solve problems—besides strengthening the quality of learning through the search for alternative ways to reach results—proposes understanding, analysis, and contact with new situations so that they can efficiently attain solutions to mathematical problems. In the geometric field, moreover, there is an additional benefit: the possibility of contextualizing knowledge with elements of reality. For many specialists, geometry is the great bridge not only between the student and reality, but also among the different conceptual fields of mathematics itself.

For many years, the teaching of algebra has dominated the Brazilian curriculum matrix in various respects. However, it has long been apparent that the absence of geometric thinking can cause large gaps in the construction of human thought. As Fainguelernt writes:

> "Geometry offers a vast field of ideas and methods of great value when it comes to the student's intellectual development, their logical reasoning, and the passage from intuition and concrete, experimental data to processes of abstraction and generalization. Geometry also facilitates the passage from the stage of concrete operations to that of abstract operations. It is, therefore, an integrating theme among the various parts of mathematics as well as a fertile field for practicing learning-by-doing and learning-to-think. It plays a primary role in teaching because intuition, formalism, abstraction, and deduction constitute its essence." (Fainguelernt, 1995, p. 45).

In a broader context, geometric thinking fosters in the student a more advanced degree of logical reasoning and intellectual growth. When well-grounded, it marks the passage from the phase of concrete thinking to a higher level of abstraction—toward working with abstract operations. However, when content is taught without establishing the proper connections, the teacher ends up interfering negatively in this transition, causing irreparable losses in learning. In this regard, problem solving emerges as a transformative tool in the educator's work concerning the approach to such content. The National Curriculum Parameters underscore that:

> "The study of geometry is a fertile field for working with problem situations and is a topic that students are naturally inclined to find interesting. Work with geometric notions contributes to learning numbers and measures, as it stimulates the student to observe, perceive similarities and differences, identify regularities, and vice versa." (Brazil, 2001, pp. 55–56).

By providing tools that ensure comprehension of other fields, geometry gains greater importance both in elementary and in secondary education. In both contexts, problem solving becomes a methodology that enhances learning, ensuring education capable of developing in the student the competencies necessary to understand space in its specificities.

RESULTS AND DISCUSSION

Amid the world of forms in which we are immersed, geometry—like mathematics—was developed from humankind's practical need to understand the surrounding universe. Its evolution represented a great leap in knowledge and gained increasing prominence through the work of major figures such as Euclid, Thales, Pythagoras, and Descartes, among others. Thus, over the years, geometry began to "give life" to human creations, and its inclusion in basic education became indispensable. Today, this branch of mathematics constitutes an organized body of knowledge indispensable to society; however, its treatment in the teaching-learning process has been losing ground to other aspects of basic mathematics, such as Algebra and Arithmetic.

It is important to emphasize that this research does not aim to raise the teaching of mathematics exclusively to geometric knowledge, but rather to analyze the contributions of geometry to the student's intellectual and cognitive development. In this sense, the approach through problem solving is the primary instructional proposal to which we refer here. This teaching methodology, which gained many adherents in the 1990s, proved to be of fundamental importance as a supporting strategy for the approach to geometry in the classroom, particularly in the context of elementary education.

Teaching from the perspective of problem solving means creating possibilities for meaningful learning that is close to the student's reality—who is increasingly immersed in spaces where the creation and mediation of conflicts, the search for swift and simplified solutions, and scientific and technological innovation are ever more prominent. Thus, based on this study, we understand the need for the teaching of mathematics—especially geometry—to overflow the limits of the classroom and take on practical meaning in each student's daily experiences.

Problem-solving methodology, in its essence, triggers the reflection–action–reflection process, defended by many authors as a key pillar in teaching and learning. Through it, it is possible to lead the student to think, organize, and understand their learning process, generating pertinent reflections on the path embarked upon. Beyond criticality—while serving as the driving force for transformation and creation of the new—thoughts, ideas, and understandings come to life through the actions performed by each student, whether inside or outside the classroom, constructing meanings pertinent to the investigative process of the classroom. Conversely, action also ensures new reflections in a continuous process of deconstruction and reconstruction that leads to learning.

It must be made clear and kept in mind that teaching geometry via problem solving goes beyond the mere application of theorems and axioms in a given context; rather, it is associated with the very act of understanding the path in order to make sense of the results found. In this sense, the teacher, as mediator of the teaching–learning process, must seek strategies that establish students' protagonism in the construction of knowledge—not only in the classroom, but also beyond it. In short, the problems and situations that will guide the process need to be endowed with information and outlines that encompass not only geometric content, but also the practical visualization of its application, modeling, abstraction, and, above all, the construction of meaning.

CONCLUSION

In its specificity, problem solving brings students closer to reality, fostering meaningful learning with significant social contributions. According to specialists in the field—for example, Dante—an education grounded in problem solving awakens in students curiosity and pleasure in learning mathematics, in addition to being a strategy that enhances understanding of the content addressed.

However, it is necessary to understand the true meaning of the term *problem solving* as applied to education. Many teachers claim to teach through this methodology, yet rely on simple repetition exercises with no stimulation of student reasoning. Thus, to ensure quality education, it is first necessary to develop in the student critical reasoning and autonomy in learning. Only in this way will education be complete and the educator have fulfilled their role.

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